# On the existence of additional solutions for equations in the $(1/2,0)\oplus(0,1/2)$ representation space*

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We analyze dispersion relations of the equations recently proposed by Ahluwalia for describing neutrino. Equations for type-II spinors are deduced on the basis of the Wigner rules for left- and right- 2-spinors and the Ryder-Burgard relation. It is shown that equations contain acausal solutions which are similar to those of the Dirac-like second-order equation. The latter is obtained in a similar way, provided that we do not apply to any constraints in the process of its deriving.

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Recently, Ahluwalia proposed the new wave equation for describing self/anti-self charge conjugate states $\lambda^{S,A}(p^\mu)$ of any spin [1]:

$$\mathcal{D}\lambda(p^\mu) = \begin{pmatrix} -\mathbb{1} & \zeta_\lambda \exp\left(\mathbf{J}\cdot\boldsymbol{\varphi}\right)\Theta_{[j]}\,\Xi_{[j]}\exp\left(\mathbf{J}\cdot\boldsymbol{\varphi}\right)\\ \zeta_\lambda \exp\left(-\mathbf{J}\cdot\boldsymbol{\varphi}\right)\Xi^{-1}_{[j]}\,\Theta_{[j]}\exp\left(-\mathbf{J}\cdot\boldsymbol{\varphi}\right) & -\mathbb{1}\end{pmatrix}\lambda(p^\mu) = 0\,. \quad (1)$$

Analogous equations for $\rho^{S,A}(p^\mu)$ bispinors have been derived in ref. [2d]. In the $j=1/2$ case spin matrices $\mathbf{J}$ are chosen to be the Pauli matrices $\boldsymbol{\sigma}/2$; in the $j=1$ case, the Barut-Muzinich-Williams matrices; $\boldsymbol{\varphi}$ are the parameters of the Lorentz boost. The notation coincides with that of refs. [1,2]. While formally the $j=1/2$ equation "may be put in the form $(\Gamma^{\mu\nu}p_\mu p_\nu + m\Gamma^\mu p_\mu - 2m^2\mathbb{1})\lambda(p^\mu)=0$ ... it turns out that $\Gamma^{\mu\nu}$ and $\Gamma^\mu$ do not transform as Poincarè tensors." Other forms of neutrino equations have been presented in refs. [3,2,4] and gauge interactions have been introduced there.[1] These constructs give alternative insights in neutrino dynamics, which could be different from that based on the common-used Weyl massless equation. Indications that neutrino may not be a Dirac particle and may have different dynamical features have appeared in analyses of the present experimental situation [5]. Earlier considerations of this problem can be found in refs. [6–9].

Both the equations (1) and equations of ref. [2,4,10] have been obtained by using different forms of the Ryder-Burgard relation [11,12,1,2,4,10] that connects zero-momentum $(0,j)$ left- and $(j,0)$ right- spinors, and the Wigner rules for their transformations to the frame with the momentum $\mathbf{p}$. The Dirac equation may also be obtained in such a way [1, footnote # 1]. The detailed discussion of this techniques can be found in [13]. It was claimed in ref. [1] that $\lambda^S(p^\mu)$ spinors answer for "*positive* energy solutions, ... [meanwhile], $\lambda^A(p^\mu)$ are the *negative* energy solutions". We, in fact, used this interpretation in [2]. Let us now check by straightforward calculations, what dispersion relations has the equation (1) in the case of $j=1/2$? Rewriting it to the form (31) of ref. [1] yields the equation of the second order in $p_0$ and the matrix in the left side has the dimension four. So, one should have eight solutions. The analytical calculation system MATEMATICA 2.2 yields that the determinant of the matrix $\mathcal{D}$ is equal to

$$\mathrm{Det}\,[\mathcal{D}] = \left(p_0^2 - p_1^2 - p_2^2 - p_3^2 - m^2\right)^2 \frac{(p_0^2 - p_1^2 - p_2^2 - p_3^2 + 3m^2 + 4mp_0)^2}{16m^4(p_0+m)^4}\quad . \qquad (2)$$

As a result of equating the determinant to zero we deduce that the equation (1) has eight solutions in total with

$$p_0 = \pm\sqrt{\mathbf{p}^{\,2} + m^2}\quad , \qquad (3)$$

each two times; and with the acausal dispersion relations:

$$p_0 = -2m \mp \sqrt{\mathbf{p}^{\,2} + m^2}\quad , \qquad (4)$$

[1]The question of equivalence of these equations still deserves further elaboration and this paper presents a certain part of this analysis.

each two times.

The same situation could be met in deriving the Dirac equation by the Ryder-Burgard-Ahluwalia technique provided that we do *not* apply to the constraint $p_0^2 - \mathbf{p}^2 = m^2$ from the beginning. Indeed,

$$\Lambda_{_R}(p^\mu \leftarrow \overset{\circ}{p}{}^\mu)\Lambda_{_L}^{-1}(p^\mu \leftarrow \overset{\circ}{p}{}^\mu) = \frac{p_0^2 + 2mp_0 + \mathbf{p}^2 + m^2 + 2(p_0+m)(\boldsymbol{\sigma}\cdot\mathbf{p})}{2m(p_0+m)} \quad , \tag{5a}$$

$$\Lambda_{_L}(p^\mu \leftarrow \overset{\circ}{p}{}^\mu)\Lambda_{_R}^{-1}(p^\mu \leftarrow \overset{\circ}{p}{}^\mu) = \frac{p_0^2 + 2mp_0 + \mathbf{p}^2 + m^2 - 2(p_0+m)(\boldsymbol{\sigma}\cdot\mathbf{p})}{2m(p_0+m)} \quad . \tag{5b}$$

Thus, the second-order momentum-representation "Dirac" equation can be written:

$$\frac{1}{2m(p_0+m)}\left[(\gamma^\mu p_\mu \mp m)\gamma^0 + 2m\right](\gamma^\nu p_\nu \mp m)\Psi_\pm(p^\mu) = 0 \quad , \tag{6}$$

or

$$\frac{1}{2m(p_0+m)}(\gamma^\nu p_\nu \mp m)\left[\gamma^0(\gamma^\mu p_\mu \mp m) + 2m\right]\Psi_\pm(p^\mu) = 0 \quad . \tag{7}$$

The corresponding coordinate-representation of these equations ($m \neq 0$ and $p_0 \neq -m$) is

$$\left[(i\gamma^\mu\partial_\mu - m)\gamma^0 + 2\wp_{u,v}m\right](i\gamma^\nu\partial_\nu - m)\Psi(x^\mu) = 0 \quad , \tag{8}$$

or

$$(i\gamma^\mu\partial_\mu - m)\left[\gamma^0(i\gamma^\mu\partial_\mu - m) + 2\wp_{u,v}m\right]\Psi(x^\mu) = 0 \quad , \tag{9}$$

where $\wp_{u,v} = \pm 1$ depending on what solutions, with either positive or negative energies, are considered.

What about the equation (1)? Can it be put in a more convenient form? The eight-component form, we proposed recently [2d,Eqs.(17,18)], does not have acausal solutions. In the process of its deriving we have assumed certain relations[2] between $\lambda^{S,A}(p^\mu)$ and $\rho^{S,A}(p^\mu)$. In the present article we are not going to apply them. Following the procedure of deriving the equations (6,7) one can arrive at the rather complicated equation:

$$\begin{aligned}\frac{1}{4m(p_0+m)}&\Big\{(\gamma^\mu p_\mu + m\gamma^0)\left[\mathcal{S}(\gamma^\nu p_\nu + m\gamma^0) - 2m\gamma^0\right] + \\ &+ \left[(\gamma^\mu p_\mu + m\gamma^0)\mathcal{S} - 2m\gamma^0\right](\gamma^\nu p_\nu + m\gamma^0)\Big\}\lambda^{S,A}(p^\mu) = 0 \quad ,\end{aligned} \tag{10}$$

where

$$\mathcal{S} = \begin{pmatrix} 0 & \zeta_\lambda\Theta\Xi \\ \zeta_\lambda\Xi^{-1}\Theta & 0\end{pmatrix} \quad . \tag{11}$$

But, as mentioned in [1], one may consider that $\phi$ in the generalized Ryder-Burgard relation (see Eq. (27) of ref. [1] or Eq. (38) in [2c]) is the azimuthal angle *associated with* $\mathbf{p}$,

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[2] See, *e.g.*, formulas (48) of ref. [1].

the 3-momentum of the particle. In this case one can find commutation relations between $\hat{p} \equiv \gamma^\mu p_\mu$, matrices $\gamma^5$, $\gamma^0$ and $\mathcal{S}$.

$$[\hat{p}, \mathcal{S}]_- = 0 \quad , \quad \left[\gamma^0, \mathcal{S}\right]_- = 0 \quad , \quad \left[\gamma^5, \mathcal{S}\right]_+ = 0 \quad , \tag{12}$$

and

$$\mathcal{S}\lambda^{S,A}(p^\mu) = \lambda^{S,A}(p^\mu) \quad , \tag{13}$$

because in this case

$$\Lambda^*_{L,R} = \Xi \Lambda_{L,R} \Xi^{-1} \quad . \tag{14}$$

We finally arrive at

$$\left[\hat{p}^2 - m^2\right] \mathbb{1}_{4\times 4}\, \lambda^{S,A}(p^\mu) = 0 \quad , \tag{15}$$

*i.e.*, at the Klein-Gordon equation for each component of $\lambda^{S,A}(p^\mu)$. Why did acausal solutions fall out? It appears bispinors $\lambda^A(p^\mu) \equiv -\gamma^5\lambda^S(p^\mu)$ can satisfy the positive-energy equation ($\zeta_\lambda = i$) and bispinors $\lambda^S \equiv -\gamma^5\lambda^A(p^\mu)$, the negative-energy one ($\zeta_\lambda = -i$), but dispersion relations will be acausal, Eq. (4), in this non-ordinary case.[3] So, assuming that in the equations (10) one should take $\zeta_\lambda = i$ for describing $\lambda^S$ and $\zeta_\lambda = -i$, for $\lambda^A$ we, in fact, implicitly impose mass-shell constraints. The same situation is for the equations (6,7), $u(p^\mu)$ and $v(p^\mu) \equiv \gamma^5 u(p^\mu)$ can satisfy both the positive- and the negative-energy equations, but the dispersion relations could be unusual.

From a mathematical viewpoint the origin of appearance of these solutions seems to be related with the properties with respect to herimitian conjugation operation of the Lorentz transformation operators, see [14, p.404] for discussion. One should further note that the problem of acausal solutions have intersections with a mathematical possible situation when operators of the continuous Lorentz transformations are combined with other transformations of the Poincarè group to give $\Lambda_R = -\Lambda_L^{-1}$. Thus, the question, whether these solutions would have some physical significance, should be solved on the basis of the rigorous analysis of the general structure of the Poincarè transformation group and of the experimental situation in neutrino physics.

Finally, let us mention that another second-order equation in the $(1/2,0)\oplus(0,1/2)$ representation space has been investigated in [15] and relations with the problem of the lepton mass spectrum have been revealed (see also [7,16]).

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[3] In the process of the proof one should take into account commutation relations (12) and hence that $\mathcal{S}^+\gamma^5\lambda^S(p^\mu) = \lambda^A(p^\mu)$ and $\mathcal{S}^-\gamma^5\lambda^A(p^\mu) = \lambda^S(p^\mu)$.

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